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Chih-Han Sah (1934–1997)

Johan L. Dupont *

Matematisk Institut, Aarhus Universitet, Ny Munkegade, DK-8000, Århus C, Denmark

During his mathematical career Han's interests have varied over a broad range of subjects: group theory, quadratic forms, rings, Riemann surfaces, algebraic topology, scissors congruences, algebraic K-theory, polylogarithms, combinatorial geometry, applications to electrical engineering, conformal quantum field theory, statistical quantum mechanics, and structures of fullerenes in chemistry.

Perhaps his best-known contributions, many of which were published in the *Journal of Pure and Applied Algebra*, are in connection with the subject of "scissors congruences" of polytopes in Euclidean, spherical, or hyperbolic n-space, a subject on which he and I were collaborators for almost 20 years.

It is an elementary fact that two polygons in the Euclidean plane with the same area are *scissors congruent*, i.e. they can be cut into finitely many pairwise congruent pieces. However, Gauss had already noticed in 1844 that the analogous statement in dimension three is far from obvious, and on the contrary Hilbert asked for a proof that it is false as the third problem on his famous list in 1900. This was immediately done by Max Dehn whose proof is nowadays usually formulated in terms of the so-called *Dehn-invariant*. The subject matter – although it has obvious generalizations – was more or less forgotten except among a few classical geometers.

Curiously Han and I both got interested in this subject by talking to the same person, Dennis Sullivan, but at different times and places. In my own case I was visiting I.H.E.S. in Paris in January 1978 when Dennis mentioned the Dehn-invariant at a lunch conversation, and said that it seemed to be a kind of homological invariant. A few years earlier Dennis had apparently made the same comment to Han thus similarly introducing him to the subject, and in 1979 Han published his book [4] in the Pitman Research Notes Series. The main result was the beautiful theorem that in all dimensions the so-called Hadwiger invariants determine Euclidean polytopes up to translational s.c. (i.e., the only congruences allowed are translations). It turned out that this theorem had already been proved (but not published) a few years before by two Danish mathematicians, B. Jessen and A. Thorup. A more important contribution of the Pitman Notes was that they pointed to the close relation between the notion of scissors conguences

* Corresponding author. E-mail: dupont mi.aau.dk

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and homological algebra, in particular the cohomology theory of groups (in this case, of the group of isometries involved).

In the same year I came across an earlier version of the manuscript. I had never heard of Chih-Han Sah before and I was rather surprised when I realized that, firstly, he and I had very similar ideas on the relation between s.c. and homology theory, and secondly, some of my results contradicted some of his! I therefore wrote a letter to him describing both the discrepancies and my own ideas on the subject. I promptly got back a very kind reply in which Han admitted that he had originally made a mistake but that it had been corrected in the printed version. At the same time he invited me for a visit to Stony Brook and so, instead of trying to compete, we started a long term collaboration. I shall not go into the details of our work, much of which is reviewed in our last paper together [3] (cf. also [1]). The main point is that the subject of s.c., which for a long time was considered somewhat exotic, is now fully integrated into modern mathematics, having close connections to well established fields such as homological algebra and algebraic K-theory, characteristic classes for flat bundles and foliations, hyperbolic 3-manifolds, and even (though more speculatively) subjects like motivic cohomology and conformal field theory.

For instance, by a classical geometric construction (going back to Gerling, a contemporary of Gauss), the s.c. group is 2-divisible also in non-Euclidean geometry, and the generalization to p-divisibility for any prime p for hyperbolic 3-space led to the proof of the first nonsolvable case (for $SL(2,\mathbb{C})$) of the so-called Friedlander–Milnor Conjecture on the homology of the discrete group underlying a Lie group (cf. [2]).

In the opposite direction, applications of well-known theorems in algebraic K-theory by Borel and Suslin have greatly clarified the structure of the s.c. group in spherical and hyperbolic 3-space, and the remaining problem is equivalent to the so-called "rigidity question" in algebraic K-theory. In particular, explicit necessary and sufficient conditions are now known for s.c. in these geometries provided the vertices of the polyhedra are defined over the field of algebraic numbers (cf. [3]).

Han was always very generous giving credits to everybody else, in particular to his collaborators, but in fact he would usually be the one to come up with the first wild idea. He always followed a principle of sharing the maximal amount of information including half-baked ideas. Especially after the introduction of e-mail our correspondence grew exponentially. He could type almost as fast as he talked and – as those who knew him will recall – he talked very fast! His energy and enthusiasm was legendary, and this is probably how most of us will remember him.

References

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